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Numerical methods

MME308

Assignment 6

Grop.

Problem no: 1,2,3,4,5

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Problem 1

Given

$$F(x)=-0.6x^2+2.4x+5.5 \qquad (1)$$

$$X_l=5 \qquad x_u=10 \qquad \acute{\varepsilon}_a=\text{less than } 5\%$$

Required

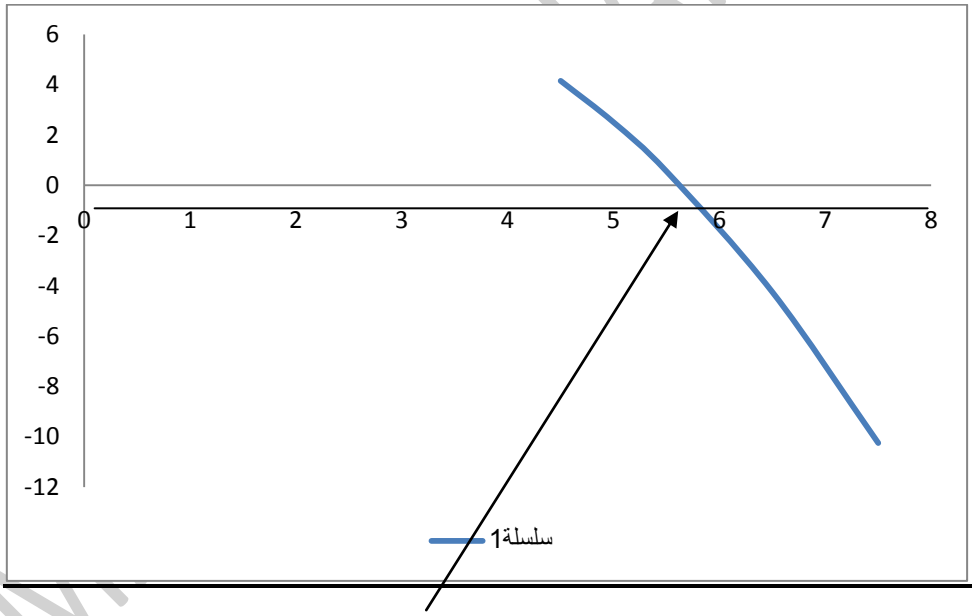
Fined the root by using:

1 --graphical technique-

2- bisection method

Solution:

1-estimte the root by using graphical technique.



The root ≈ 5.6

2- Estimate the root By using bisection method.

$$X_r = \frac{xl-xu}{2} \qquad X_r= \frac{5+10}{2} =7.5$$

Substituting in eq, (1) by x_r

$$F(X_r)= -10.25$$

$$\text{If } f(X_l)f(X_r) \quad 0; \qquad X_l=X_l \text{ and } X_u=X_r$$

If $f(X_l)f(X_r)$ 0; $X_l=X_r$ and $X_u=X_u$

iteration	x_i	$f(x_i)$	x_u	$f(x_u)$	x_r	$f(x_r)$	$\acute{\varepsilon}_a$
1	5.0000	2.5000	10.0000	-30.5000	7.5000	-10.2500	
2	5.0000	2.5000	7.5000	-10.2500	6.2500	-2.9375	20.00
3	5.0000	2.5000	6.2500	-2.9375	5.6250	0.0156	11.11
4	5.6250	0.0156	6.2500	-2.9375	5.9375	-1.4023	5.26
5	5.6250	0.0156	5.9375	-1.4023	5.7813	-0.6787	2.70

Problem 2

Given

$$F(x)=-13-20x+19x^2-3x^3 \qquad (2)$$

$$X_l=-1 \qquad x_u=0 \qquad \acute{\varepsilon}_a=\text{less than } 2\%$$

Required

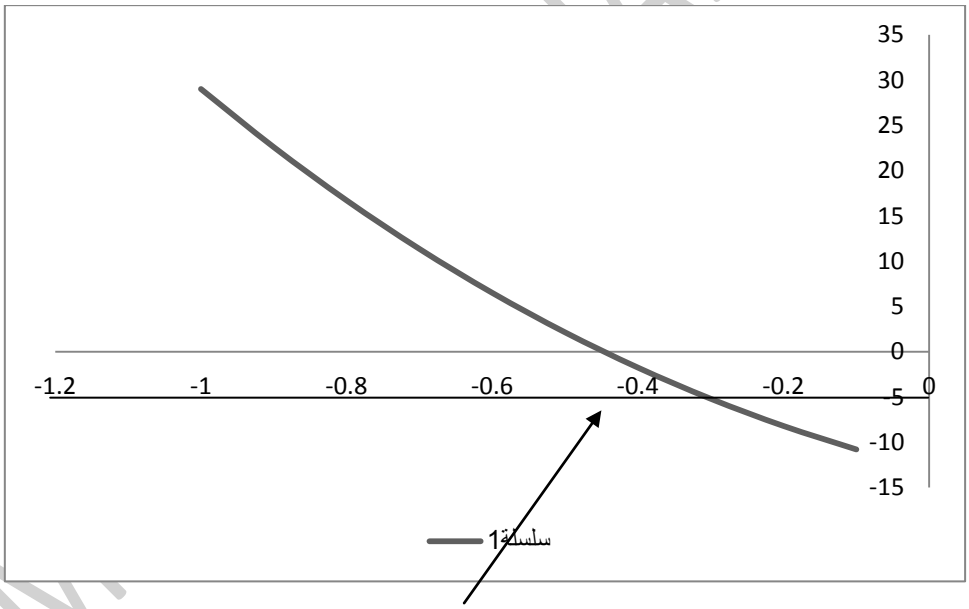
Fined the root by using:

1 --graphical technique-

2- bisection method

Solution

1-Find the root by using graphical technique.



The root ≈-0.46

2-Fined the root by using bisection method.

$$X_r = \frac{xl-xu}{2} \qquad X_r= \frac{-1+0}{2} =-0.5$$

Substituting in eq, (2) by x_r

$$F(X_r)= 2.125$$

If $f(X_l)f(X_r)$ 0; $X_l=X_l$ and $X_u=X_r$

If $f(X_l)f(X_r)$ 0; $X_l=X_l$ and $X_u=X_r$

i	x_l	$f(x_l)$	x_u	$f(x_u)$	x_r	$f(x_r)$	$\acute{\varepsilon}_a$
1	-1.0000	29.0000	0.0000	-13.0000	-0.5000	2.1250	
2	-0.5000	2.1250	0.0000	-13.0000	-0.2500	-6.7656	100.00
3	-0.5000	2.1250	-0.2500	-6.7656	-0.3750	-2.6699	33.33
4	-0.5000	2.1250	-0.3750	-2.6699	-0.4375	-0.3621	14.29
5	-0.5000	2.1250	-0.4375	-0.3621	-0.4688	0.8588	6.67
6	-0.4687	0.8568	-0.4375	-0.3621	-0.4531	0.2418	3.45
7	-0.4531	0.2418	-0.4375	-0.3621	-0.4453	-0.0616	1.75

Problem 3

Given

$$\mathbf{Sin(X) + Cos(X) - 0.2 = 0 \qquad (3)}$$

$$\mathbf{´E \text{ less than } 2\%}$$

Required

Positive solution that is closest to the origin (X=0)

Solution

1-By using fixed point iteration :

Rewriting the eq (3) by addition x from two side as :x=g(x)

$$\mathbf{X = X + \sin(x) + \cos(x) - 0.2}$$

select Initial guess $\boldsymbol{X_0 = 0}$

$$\mathbf{X_{i-1} = X_i + \sin(x_i) + \cos(x_i) - 0.2}$$

-At substation in the sin(x), cos(x) should by converting from degrees to radians.

- $\mathbf{X_1 = X_0 + \sin(x_0) + \cos(x_0) - 0.2 = 0 + \sin(0*180/pi) + \cos(0*180/pi) - 0.2}$

$X_1 = 0.8000$

- $\mathbf{X_2 = 0.8000 + \sin(0.8*180/pi) + \cos(0.8*180/pi) - 0.2}$

X2= 2.0141

- $\mathbf{X_3 = 2.0141 + \sin(2.0141*180/pi) + \cos(2.0141*180/pi) - 0.2}$

X3=2.2885

- $\mathbf{X_4 = 2.2885 + \sin(2.2885*180/pi) + \cos(2.2885*180/pi) - 0.2}$

$X_4 = 2.1842$

Complete The Solution In Table

i	xi	g(x)	e
0	0.0000	0.8000	
1	0.8000	2.0141	100
2	2.0141	2.2885	60.2793
3	2.2885	2.1842	11.9930
4	2.1842	2.2263	4.7787
5	2.2263	2.2095	1.8914

In this the root at error less than 2% X_5=2.2263

Check by substituting your final answer into the original eq (3)

2.2263+sin(127.6223)+cos(127.6223)-0.2=2.2078

2-By using bisection method :

The max angle can be used 360^0.

i	xi	f(xi)	x_u	$f(x_u)$	x_m	$f(x_m)$	ε
1	0	0.8	360	0.8	180	-1.2	
2	0	0.8	180	-1.2	90	0.8	100
3	90	0.8	180	-1.2	135	-0.2	33.333
4	90	0.3411	135	-0.2	112.5	0.3411	20.
5	112.5	0.0758	135	-0.2	123.75	0.0758	9.09
6	123.75	0.0758	135	-0.2	129.375	-0.0613	4.347
7	123.75	0.0075	129.375	-0.0613	126.5625	0.0075	2.22
8	126.5625	0.0075	129.375	-0.0613	127.9688	-0.0268	1.098

127.9688*pi/180=2.2323

Problem 4

Given

$$F(x)=8e^{-x}\sin(X) - 1 \qquad (4)$$

Required

Fined the root by using:

1 – Newton- Raphson method

2- secant method

Solution

1-By using Newton- Raphson method

$$X_{I+1} = x_I - \frac{f(xI)}{f`(XI)}$$

The first derivative of the function can be evaluated as

$$F`(X)=8e^{-x}\cos(X) - 8e^{-x}\sin(X) \qquad (4)`$$

Initial iteration: $X_0=0.3$

Iteration 1:

Substituting in eq, (4) and (4)` by x_0

$$F(x_0)=0.7514 \qquad F`(x_0)=3.9104$$

$$X_1 = x_0 - \frac{f(x0)}{f`(X0)}$$

$$X_1 = 0.3 - \frac{0.7514}{3.9104} = 0.1078$$

Iteration 2:

$$F(x_1)=-0.227 \qquad F`(x_1)=4.7389$$

$$X_2 = x_1 - \frac{f(x1)}{f`(X1)}$$

$$X_2 = 0.1078 - \frac{-0.227}{4.7389} = 0.1557$$

Iteration 3:

$$F(x_2)=0.0619 \qquad F`(x_2)=4.5173$$

$$X_3 = x_2 - \frac{f(x2)}{f`(X2)}$$

$$X_3 = 0.1557 - \frac{0.0619}{4.5173} = 0.1420$$

Iteration 4:

$$F(x_3)=-0.0175 \qquad F`(x_3)=4.5796$$

$$X_4 = x_3 - \frac{f(x3)}{f`(X3)}$$

$$X_4 = 0.1420 - \frac{0.0175}{4.5796} = 0.1459$$

i	xi	f(xi)	f`(xi)	ɛa
0	0.3000	0.7514	3.9104	
1	0.1078	-0.2270	4.7389	-1.7818
2	0.1557	0.0619	4.5173	0.3075
3	0.1420	-0.0175	4.5796	-0.0965
4	0.1459	0.0049	4.5621	0.0262

The root at use four iteration **= 0.1459**

Check by substituting your final answer into the original eq (4) by x4

But should be convert the x_4 from degrees to radians:

=0.1459*180/pi=8.3637^0

$$F(x)=8e^{-0.1459}\sin(8.3637)-1=0.0005$$

2-By using secant method

$$X_{i+1}=x_i - \frac{(xi-xi-1)f(xi-1)}{f(xi)-f(xi-1)}$$

Iteration 1: $X_0=0.4 \quad x_{-1}=0.5$

$$F(X_{-1})=8e^{-0.5} *\sin(0.5*180/pi) - 1 = 1.3263$$

$$F(X_0)=8e^{-0.4} *\sin(0.4*180/pi) - 1 = 1.0883$$

$$X_1=X_0 - \frac{(x0-x-1)f(x0)}{f(x0)-f(x-1)} =-0.0572$$

Iteration 2:

$$F(x0) = 1.0883 \qquad f(x1) = -1.4846$$

$$X_2=X_1 - \frac{(x1-x\,0)f(x1)}{f(x1)-f(x0)} =0.2060$$

Iteration 3:

$$F(x1) = -1.4846 \qquad f(x2) = 0.3347$$

$$X_3=X_2 - \frac{(x2-x1)f(x2)}{f(x2)-f(x1)} =0.1581$$

Iteration 4:

$$F(x2) = 0.3347 \qquad f(x3) = 0.0751$$

$$X_4=X_0 - \frac{(x3-x\,2)f(x3)}{f(x3)-f(x\,2)} =1440$$

Iteration 5:

$$F(x3) = 0.0751 \qquad f(x4) = -0.0058$$

$$X_5=X_0 - \frac{(x4-x\,3)f(x4)}{f(x4)-f(x\,3)} =0.1450$$

i	xi-1	f(xi-1)	xi	f(xi)	ɛa
0	0.5000	1.3263	0.4000	1.0883	
1	0.4000	1.0883	-0.0572	-1.4846	-0.4572
2	-0.0572	-1.4846	0.2066	0.3347	0.2638
3	0.2066	0.3347	0.1581	0.0751	-0.0485
4	0.1581	0.0751	0.1440	-0.0058	-0.0140

The root at use four iteration **= 0.1581**

Check by substituting your final answer into the original eq (4) by x4

But should be convert the x_4 from degrees to radians:

=0.1581*180/pi=9.0584^0

$$F(x)=8e^{-0.1581}\sin(9.0584)-1=0.0753$$

Problem 5

Given

F(X)=$e^{1/x}$-x **Initial guesses [2]**

Required

Find the root By using Newton- Raphson method

Solution

$$X_{i+1}=x_i-\frac{f(xi)}{f`(Xi)}$$

The first derivative of the function can be evaluated as

$$F`(x)=(-1/X^2 * e^{1/x}-1)$$

Initial iteration X_0 = 2

Iteration 1:

$$F(x_0)=-0.3513 \qquad F`(x_0)=-1.4122$$

$$X_1 = x_0 - \frac{f(x0)}{f`(X0)}$$

$$X_1 = 2 - \frac{-0.3513}{-1.4122} = 1.7513$$

Iteration 2:

$$F(x_1)=0.0188 \qquad F`(x_1)=-1.5772$$

$$X_2 = x_1 - \frac{f(x1)}{f`(X1)}$$

$$X_2 = 1.7513 - \frac{0.0188}{-1.5772} = 1.7632$$

Iteration 3:

$$F(x_2)=0.0001 \qquad F`(x_2)=-1.5672$$

$$X_3 = x_2 - \frac{f(x2)}{f`(X2)}$$

$$X_3 = 1.7632 - \frac{0.0001}{-1.5672} = 1.7632$$

i	xi	f(xi)	f`(xi)	ε
0	2	-0.3513	-1.4122	
1	1.7513	0.0188	-1.5772	14.2041
2	1.7632	0.0001	-1.5672	0.6768
3	1.7632	0.0000	-1.5671	0.0022

The solution at the error less than 2% =1.7632

Check by substituting your final answer into the original eq (5) by X_3

$$F(x) = e^{1/1.7632} - 1.7632 = 0.0000$$